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ON THE SOLUTION OF HEAT TRANSFER THROUGH AN ARRAY OF EXTENDED SURFACES

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Abstract—A general method of analysis is presented for heat transfer through an array of extended surfaces containing any number of fins. By treating each fin in the array as a finite element whose characteristics are to be determined by the solution of a one-dimensional single fin problem, the analysis of the problem is reduced to the solution of a system of algebraic equations. In this system, the elements of the coefficient matrix are related to the fundamental solutions of the fin equation. Explicit expressions are presented for the determination of the elements of the coefficient matrix. The application is illustrated with an example.

NOMENCLATURE

$A(x), A_0$	cross-sectional area, and reference cross-sectional area, respectively
c	defined by equation (5b)
$h, h(x)$	heat transfer coefficients
$K(X)$	defined by equation (5a)
$k, k(x)$	thermal conductivities
$k_{ij}^{(e)}$	coefficient in the fin matrix for element 'e' having nodes identified by i and j
L_0	reference length
p, p_0	perimeter, and perimeter of reference cross-section, respectively
$Q_i^{(e)}$	heat flow rate into element 'e' through the node i
$S(x)$	lateral surface area
$T(x)$	temperature at location x
T_i	temperature at node i
T_∞	temperature of the surrounding fluid
$u(X), v(X)$	two fundamental solutions of the fin
$U(X), V(X)$	equation solutions defined by equations (9d) and (9e)
$W(X)$	defined by equation (5b)
x	axial coordinate
X	dimensionless axial coordinate, x/L_0.

Greek symbols

θ	temperature in excess of the surrounding fluid temperature
κ	defined by equation (16e).

Subscripts

b	fin base
t	fin tip.

INTRODUCTION

THE HEAT transfer analysis of a single fin has been extensively studied in the literature and the solutions for various fin geometries are well documented [1, 2]. Such results are applicable to study heat flow through a finned matrix whose repeating section is a single fin. In many engineering applications such as those encountered in certain types of compact heat exchangers, the repeating section of a finned matrix is not a single fin, but is an array of fins. Consider for illustration purposes a complex finned matrix in the coolant passage between two hot surfaces as shown in Fig. 1(a). Let the temperature of the two hot surfaces be uniform and equal. Then from symmetry, the heat transfer characteristics of this fin matrix can be represented with that of the 'repeating section' enclosed by the dashed lines. This repeating section, as shown in Fig. 1(b), is not a single fin, but is an array of fins.

The heat transfer analysis of an array of fins is a complicated matter; only a limited number of papers is available on this subject. The method of analysis proposed in ref. [2] involves a simultaneous solution of a set of ordinary differential equations associated with each of the fins in the array. If the differential equations can be integrated, the problem becomes one of solving a system of linear algebraic equations associated with the determination of the integration constants. The application of the method for the solution of practical problems is quite involved. To alleviate this difficulty, a procedure is described in ref. [3], which treats each fin as a lumped parameter, and cascades all the fins in the array via matrix operations. In a previous paper [4] the authors briefly mentioned that such problems could readily be solved by using the well-known finite element assembly procedure.

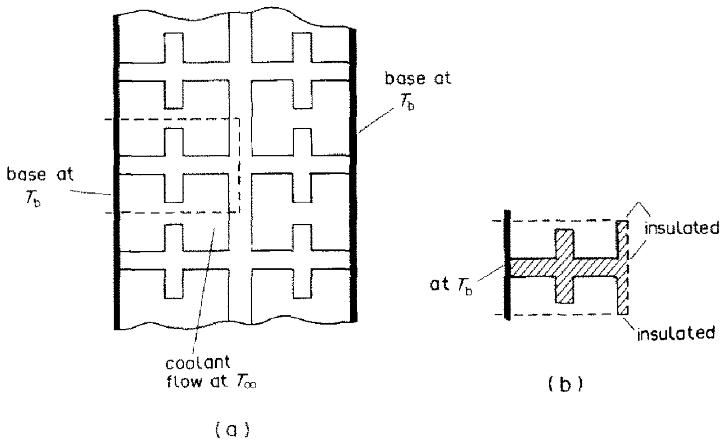

FIG. 1. (a) Coolant passage with complex fin array. (b) The repeating section of the fin array.

The present paper describes a straightforward procedure for heat transfer analysis of an array of extended surfaces. In this method, each fin in the array is considered as a finite element whose characteristics are determined by the solution of a one-dimensional (1-D) fin problem. Thus the analysis of the problem is reduced to the solution of a system of algebraic equations. In such a system, each element of the *global fin matrix* (i.e. coefficient matrix) is related to the fundamental solutions of the ordinary differential equation governing heat transfer through a single fin. Explicit expressions are developed for the determination of the elements of this matrix.

THE DEFINING ELEMENTS AND THEIR PROPERTIES

The problem of an array of fins can be split into a series of simpler fin problems if one considers each fin as a finite element in the assembly whose characteristics can be determined by the basic law of heat conduction.

The 1-D fin equation for a single fin of variable cross-section can be written in the form [5]

$$\frac{d}{dx}\left(A(x)\frac{dT_f(x)}{dx}\right)-\frac{h(x)}{k}\frac{dS(x)}{dx}(T_f(x)-T_\infty)=0, \quad (1)$$

which holds from the fin base at $x = x_b$ to the fin tip at $x = x_t$. Here, $A(x)$, $h(x)$, $S(x)$ and $T_f(x)$ are, respectively, the fin cross-sectional area, the heat transfer coefficient, the fin perimeter and the fin temperature at the location x; and T_∞ is the temperature of the ambient.

For convenience in the analysis, we introduce the following dimensionless variables

$$X=\frac{x}{L_0},\quad \theta(X)=\frac{T_f(x)-T_\infty}{\Delta T}, \quad (2a,b)$$

where L_0 is a reference length and ΔT is a reference temperature difference. Then, equation (1) can be written in the form

$$\frac{d}{dX}\left(K(X)\frac{d\theta(X)}{dX}\right)-M^2W(X)\theta(X)=0,$$

$$\text{in } X_b < X < X_t, \quad (3)$$

where the following additional dimensionless quantities are defined as

$$K(X)=\frac{A(x)}{A_0},\quad M^2=\frac{h_{av}p_0}{kA_0}L_0^2, \quad (4a,b)$$

$$p(x)=\frac{dS(x)}{dx},\quad W(X)=\frac{h(x)}{h_{av}}\frac{p(x)}{p_0}. \quad (4c,d)$$

Here, A_0 is a reference cross-section, h_{av} an average heat transfer coefficient and p_0 the perimeter of the reference cross-section A_0. As pointed out by Gardner [1], equation (4c) is valid for thin fins and spins; that is when the square of the slope of the fin sides is negligible compared to unity.

Generalized fundamental solution

We now examine the fundamental solutions of equation (3) for the following sufficiently general situation in which the functions $K(X)$ and $W(X)$ are chosen as

$$K(X)=X^{1-2m},\quad W(X)=c^2n^2X^{2c-2}K(X). \quad (5a,b)$$

Clearly, the numerical values of the coefficients c, n and m establish the specific forms of the functions $K(X)$ and $W(X)$. For example, some special cases of $K(X)$ and $W(X)$, with $cn = 1$ are presented in Table 1.

Introducing equations (5a) and (5b) into equation (3), the fin equation takes the form

$$\frac{d^2\theta(X)}{dX^2}+\frac{1-2m}{X}\frac{d\theta(X)}{dX}-M^2n^2c^2X^{2c-2}\theta(X)=0, \quad (6)$$

which is now a special case of the generalized Bessel equation.

The two fundamental solutions of equation (6) are obtained as

$$u(X)=X^mI_{m/c}(nMX^c), \quad (7a)$$

and

$$v(X)=X^mK_{m/c}(nMX^c), \quad (7b)$$

or

$$v(X)=X^mI_{-m/c}(nMX^c),\quad \text{for } m/c \text{ nonintegral.} \quad (7c)$$

Table 1. Values of c, m and n for some special cases of $K(X)$ and $W(X)$ with $cn = 1$

	$K(X)$	$W(X)$	m	c	n	m/c
Longitudinal fin of rectangular profile	1	1	$\frac{1}{2}$	1	1	$\frac{1}{2}$
Longitudinal fin of triangular profile	X	1	0	$\frac{1}{2}$	2	0
Longitudinal fin of concave parabolic profile	X^2	1	$-\frac{1}{2}$	0	∞	$-\infty$
Longitudinal fin of convex parabolic profile	$X^{1/2}$	1	$\frac{1}{4}$	3/4	4/3	1/3
Radial fin of rectangular profile	X	X	0	1	1	0
Radial fin of hyperbolic profile	1	X	$\frac{1}{2}$	3/2	2/3	1/3
Conical spine	X^2	X	$-\frac{1}{2}$	$\frac{1}{2}$	2	-1
Spine fin of concave parabolic profile	X^4	X^2	$-3/2$	0	∞	$-\infty$
Spine of convex parabolic profile	X	$X^{1/2}$	0	3/4	4/3	0

For the special case of $cn = 1$ and $c = 0$, the fin equation (6) reduces to the Euler equation. Then the two fundamental solutions in any interval not containing the origin are

$$u(X) = X^{r_1}, \quad v(X) = X^{r_2}, \tag{8a,b}$$

where

$$r_k = m+(-1)^{1+k}(m^2+M^2)^{1/2}, \quad k = 1,2. \tag{8c}$$

The solution for $\theta(X)$ can be constructed by taking any linear combination of the two fundamental solutions $u(X)$ and $v(X)$; we prefer to construct the solution in the form

$$\theta(X) = \theta_b U(X)+\theta_t V(X), \tag{9a}$$

where

$$\theta_b = \theta(X_b) \quad \text{and} \quad \theta_t = \theta(X_t). \tag{9b,c}$$

Then the functions $U(X)$ and $V(X)$, satisfying the requirement of equations (9b) and (9c) are determined as

$$U(X) = \frac{u(X)v(X_t)-u(X_t)v(X)}{u(X_b)v(X_t)-u(X_t)v(X_b)}, \tag{9d}$$

$$V(X) = \frac{u(X_b)v(X)-u(X)v(X_b)}{u(X_b)v(X_t)-u(X_t)v(X_b)}, \tag{9e}$$

We note that with the choice of the functions $U(X)$ and $V(X)$ as defined by equations (9d) and (9e), the following conditions are automatically satisfied

$$U(X_b) = 1, \quad U(X_t) = 0, \tag{10a}$$

$$V(X_b) = 0, \quad V(X_t) = 1. \tag{10b}$$

In the case of a single fin problem, the integration constants θ_b and θ_t are determined by constraining the solution (9a) to meet the two boundary conditions specified at $X = X_b$ and $X = X_t$, and the analysis of simple and straightforward.

In the case of a fin array in which several fins are connected, it becomes an elaborate and complicated matter to solve the problem by writing equation (3) for each of the fins in the array and then try to determine the resulting integration constants by matching the solutions to meet the boundary conditions. To alleviate such difficulty we now describe an efficient method of solving the heat transfer problems for a fin array.

Consider the 1-D heat flow through a typical element of the fin array as shown in Fig. 2. Let Q_b and Q_t be the heat flow rates entering the element at the node $X = X_b$ and $X = X_t$ through the surfaces A_b and A_t, respectively. Let the temperature distribution $\theta(X)$ in the element be governed by equation (9a). The heat flow rates Q_b and Q_t entering the elements are determined according to Fourier's law, and the result is written in matrix notation as

$$kA_0\frac{\Delta T}{L_0}\begin{bmatrix} -K(X_b)U'(X_b) & -K(X_b)V'(X_b) \\ K(X_t)U'(X_t) & K(X_t)V'(X_t) \end{bmatrix}\begin{Bmatrix}\theta_b \\ \theta_t\end{Bmatrix} = \begin{Bmatrix} Q_b \\ Q_t \end{Bmatrix}, \tag{11}$$

where primes denote differentiation with respect to X.

Equation (11) is written in the 'standard form' for the finite element method [6]. When the temperature of the surrounding fluid T_∞ is not constant and therefore has different values for the fins of the array, equation (11) must be written in the form

$$\begin{bmatrix} k_{bb} & k_{bt} \\ k_{tb} & k_{tt} \end{bmatrix}\begin{Bmatrix} T_b \\ T_t \end{Bmatrix} = \begin{Bmatrix} Q_b \\ Q_t \end{Bmatrix} + \begin{Bmatrix} p_b \\ p_t \end{Bmatrix}, \tag{12a}$$

or more compactly in the form

$$[K]\{T\} = \{Q\}+\{P\}, \tag{12b}$$

where $\{T\}$ is the column vector of the *two node temperatures*, $\{Q\}$ is the column vector of the *two node heat transfer rates* and $[K]$ is the *fin matrix* of thermal influence coefficients:

$$k_{bb} = -(k/L_0)A_0K(X_b)U'(X_b), \tag{13a}$$

$$k_{bt} = -(k/L_0)A_0K(X_b)V'(X_b), \tag{13b}$$

$$k_{tb} = (k/L_0)A_0K(X_t)U'(X_t), \tag{13c}$$

$$k_{tt} = (k/L_0)A_0K(X_t)V'(X_t), \tag{13d}$$

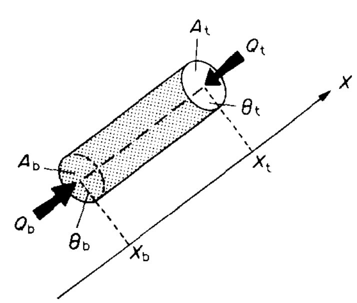

FIG. 2. A schematic representation of an element of the fin array.

and $\{P\}$ is the column vector of the two components

$$P_b = (k_{bb} + k_{bt})T_\infty/\Delta T, \tag{14a}$$

$$P_t = (k_{tb} + k_{tt})T_\infty/\Delta T. \tag{14b}$$

For the fin equation defined by equation (6), the functional forms of $U'(X)$ and $V'(X)$ can be determined by introducing the fundamental solutions, equations (7a) and (7b) into equations (9d) and (9e) and then substituting the resulting expressions into equations (13a)–(13d). Then, when (m/c) *is integral*, equations (13a)–(13d) become:

$$k_{bb} = -\kappa n M X_b^{c-2m} N(nMX_b^c, nMX_t^c)/D, \tag{15a}$$

$$k_{bt} = \kappa/(D X_b^m X_t^m), \tag{15b}$$

$$k_{tb} = \kappa/(D X_b^m X_t^m), \tag{15c}$$

$$k_{tt} = -\kappa n M X_t^{c-2m} N(nMX_t^c, nMX_b^c)/D, \tag{15d}$$

where

$$\kappa = (k/L_0)A_0 c, \tag{15e}$$

$$N(X, Y) = I_{m/c-1}(X)K_{m/c}(Y) + I_{m/c}(Y)K_{m/c-1}(X), \tag{15f}$$

$$D = I_{m/c}(nMX_b^c)K_{m/c}(nMX_t^c) - I_{m/c}(nMX_t^c)K_{m/c}(nMX_b^c), \tag{15g}$$

and in equation (15f), X and Y are dummy variables. When (m/c) *is nonintegral*, equations (13a)–(13d) take the form:

$$k_{bb} = -\kappa n M X_b^{c-2m} N(nMX_b^c, nMX_t^c)/D, \tag{16a}$$

$$k_{bt} = -\frac{2}{\pi}\kappa \sin\left[\left(\frac{m}{c}-1\right)\pi\right] \Big/ (D X_b^m X_t^m), \tag{16b}$$

$$k_{tb} = -\frac{2}{\pi}\kappa \sin\left[\left(\frac{m}{c}-1\right)\pi\right] \Big/ (D X_b^m X_t^m), \tag{16c}$$

$$k_{tt} = -\kappa n M X_t^{c-2m} N(nMX_t^c, nMX_b^c)/D, \tag{16d}$$

where

$$\kappa = (k/L_0)A_0 c, \tag{16e}$$

$$N(X, Y) = I_{m/c-1}(X)I_{-m/c}(Y) - I_{m/c}(Y)I_{m/c+1}(X), \tag{16f}$$

$$D = I_{m/c}(nMX_b^c)I_{-m/c}(nMX_t^c) - I_{m/c}(nMX_t^c)I_{-m/c}(nMX_b^c), \tag{16g}$$

and in equation (16f), X and Y are dummy variables.

For the special case of $cn = 1$ and $c = 0$, we recall that the fin equation (6) reduces to the Euler equation and its two fundamental solutions are given by equations (8a) and (8b). In that case, equations (8a) and (8b) are introduced into equations (9d) and (9e) and the resulting expressions are substituted into equations (13a)–(13d), which become

$$k_{bb} = -\kappa N(X_b, X_t)/D, \tag{17a}$$

$$k_{bt} = \kappa(r_1 - r_2)/D, \tag{17b}$$

$$k_{tb} = \kappa(r_1 - r_2)/D, \tag{17c}$$

$$k_{tt} = -\kappa N(X_t, X_b)/D, \tag{17d}$$

where

$$\kappa = (k/L_0)A_0, \tag{17e}$$

$$N(X, Y) = (r_1 X^{r_1} Y^{r_2} - r_2 Y^{r_1} X^{r_2})/X^{2m}, \tag{17f}$$

$$D = X_b^{r_1} X_t^{r_2} - X_t^{r_1} X_b^{r_2}, \tag{17g}$$

and in equation (17f), X and Y are dummy variables. The foregoing procedure establishes the fundamental properties of each element (i.e. fin) in a system composed of many individual elements connected through the nodes. The global relationships between the individual elements for the entire system can now be established by utilizing the finite element assembly procedure described below.

FINITE ELEMENT ASSEMBLY PROCEDURE

Let us consider the heat flow network system composed of many individual fins ($e = 1, 2, \ldots, E$), connected through the nodes ($n = 1, 2, \ldots, N$). We focus our attention on an individual fin treated as a finite element similar to the one shown in Fig. 2, except that the end points are denoted as the nodes m and n, and the fin is referred to as the element 'e' of the assembly. Let T_m and T_n be the temperatures, and $Q_m^{(e)}$ and $Q_n^{(e)}$ be the heat flow rates entering the element 'e' through the nodes m and n, respectively. The relationship given by equation (12b) is now applied to this element, and written in the abbreviated form as

$$[K^{(e)}]\{T\} = \{Q^{(e)}\} + \{P^{(e)}\}, \tag{18a}$$

or in the expanded form as

$$\begin{bmatrix} 0 & \cdot & 0 & \cdot & 0 & \cdot & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & k_{nn}^{(e)} & \cdot & 0 & \cdot & k_{nm}^{(e)} & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & \cdot & 0 & \cdot & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & k_{mn}^{(e)} & \cdot & 0 & \cdot & k_{mm}^{(e)} & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & 0 & \cdot & 0 & \cdot & 0 & \cdot & 0 \end{bmatrix} \begin{Bmatrix} T_1 \\ \cdot \\ T_n \\ \cdot \\ \cdot \\ \cdot \\ T_m \\ \cdot \\ T_N \end{Bmatrix} = \begin{Bmatrix} 0 \\ \cdot \\ Q_n^{(e)} \\ \cdot \\ 0 \\ \cdot \\ Q_m^{(e)} \\ \cdot \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ \cdot \\ P_n^{(e)} \\ \cdot \\ 0 \\ \cdot \\ P_m^{(e)} \\ \cdot \\ 0 \end{Bmatrix}. \tag{18b}$$

We note that: (1) at the nodes where the fins are connected the temperature is the same for all fins forming the node, and (2) the algebraic sum of the heat flow rates at each node must be equal to the external heat flow at that node.

Now, equation (18b) is written for each element and the results are summed up for all $e = 1, 2, \ldots, E$, where E is the total number of elements in the assembly. We obtain

$$[K]\{T\} = \{Q\} + \{P\}, \quad (19a)$$

where $\{Q\}$ is a column vector for external nodal heat flow rates, and the global matrix $[K]$ and the column vector $\{P\}$ are given by

$$[K] = \sum_{e=1}^{E} K^{(e)}, \quad (19b)$$

$$\{P\} = \sum_{e=1}^{E} P^{(e)}. \quad (19c)$$

The computer assembly procedure forming the global matrix $[K]$ consists of the following steps: (1) Set up $N \times N$ null master matrix (all zero entries); (2) Starting with one element, insert its coefficients $k_{nn}^{(e)}$, $k_{nm}^{(e)}$, $k_{mn}^{(e)}$ and $k_{mm}^{(e)}$ into the master matrix in the locations designated by their indices. Each time a term is placed in a location where another term has already been placed, it is added to whatever value is there; (3) Return to step 2 and repeat this procedure for one element after another until all elements have been treated. The result will be the global matrix $[K]$. This assembly procedure is an essential part of the finite element method.

The boundary conditions

The next step in the analysis is the inclusion of the boundary condition at each node into the problem.

At each node a boundary condition can be written as

$$A_n T_n + B_n Q_n = F_n, \quad n = 1, 2, \ldots, N, \quad (20a)$$

where Q_n is the external nodal heat flow rate; A_n, B_n and F_n are known constants. The cases $A_n = 1$, $B_n = 0$ and $A_n = 0$, $B_n = 1$ correspond to a prescribed temperature and heat flow rate, respectively.

To preserve the sparse, banded and symmetric properties of the global matrix $[K]$ the boundary conditions, equation (20a), are rewritten as

$$(A_n/B_n)T_n + Q_n = F_n/B_n, \quad n = 1, 2, \ldots, N. \quad (20b)$$

To insert the prescribed nodal temperature $A_n = 1$, B_n is replaced by a small number, say 1×10^{-15}. Then equation (13b) expresses the fact that $T_n = F_n$ since $1 \times 10^{15} \gg 1$.

The nodal boundary conditions, equation (20b), can be written in the matrix form as

$$[A/B]\{T\} + \{Q\} = \{F/B\}, \quad (20c)$$

where

$$[A/B] \equiv \begin{bmatrix} A_1/B_1 & & & \\ & A_2/B_2 & & \\ & & \ddots & \\ & & & A_n/B_n \end{bmatrix}$$

and

$$\{F/B\} \equiv \begin{Bmatrix} F_1/B_1 \\ F_2/B_2 \\ \vdots \\ F_n/B_n \end{Bmatrix} \quad (20d,e)$$

and the elements of $[A/B]$ not shown are zero.

Finally, equations (19a) and (20c) are combined to obtain the solution in the form

$$\{T\} = ([A/B] + [K])^{-1}(\{F/B\} + \{P\}). \quad (21)$$

ILLUSTRATIVE EXAMPLE

Consider now as an illustrative example the fin matrix configuration of Fig. 3(a) to represent the repeating section of a plate fin compact heat exchanger. Let elements ② and ③ represent the splitter plates; then there is no heat transfer at the nodes and we have $Q_2 = Q_4 = 0$. Nodes '1' and '5' are subjected to prescribed heat flow rates, hence Q_1 and Q_5 are known. At node '3' there is no externally applied heat flow.

The specific fin geometry considered for this example is a plate fin as shown in Fig. 3(b), for which the standard fin equation applies, i.e.

$$\frac{d^2\theta(X)}{dX^2} - M^2\theta(X) = 0, \quad (22)$$

where $\theta(X)$ is the fin temperature in excess of the surrounding coolant temperature.

A comparison of equation (22) with equation (3)

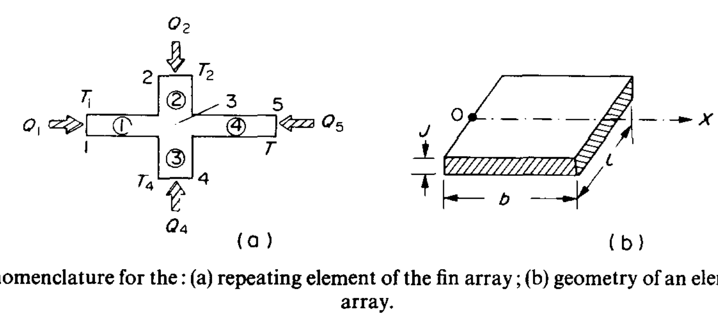

FIG. 3. The nomenclature for the: (a) repeating element of the fin array; (b) geometry of an element of the fin array.

reveals that

$$K(X) = 1 \quad \text{and} \quad W(X) = 1, \tag{23a}$$

and in view of the definitions, equations (5a) and (5b), we have

$$m = \tfrac{1}{2}, \quad c = 1 \quad \text{and} \quad n = 1. \tag{23b}$$

Then, the thermal influence coefficients, equations (16a)–(16g) reduce to

$$k_{bb} = k_{tt} = \frac{k}{L_0} A_0 M \frac{\cosh [M(X_t - X_b)]}{\sinh [M(X_t - X_b)]}, \tag{24a}$$

$$k_{bt} = k_{tb} = \frac{k}{L_0} A_0 M \frac{1}{\sinh [M(X_t - X_b)]}, \tag{24b}$$

where we utilized the relationships

$$I_{-1/2}(z) = \sqrt{(2/\pi z)} \cosh z, \quad I_{1/2}(z) = \sqrt{(2/\pi z)} \sinh z. \tag{25}$$

As a reference length we select $L_0 = b$ [see Fig. 3(b)]. The cross-sectional area is $A_0 = \delta l$. Finally we select

$$X_b = 0, \qquad X_t = 1,$$

$$M = mb \quad \text{where} \quad m = \left(\frac{2h}{k\delta}\right)^{1/2}. \tag{26}$$

Then the resulting fin matrix becomes

$$\begin{vmatrix} k_{bb}^{(e)} & k_{tb}^{(e)} \\ k_{bt}^{(e)} & k_{tt}^{(e)} \end{vmatrix} = \frac{(k\delta l m)^{(e)}}{\sinh [m^{(e)} b^{(e)}]} \times \begin{vmatrix} \cosh [m^{(e)} b^{(e)}] & -1 \\ -1 & \cosh [m^{(e)} b^{(e)}] \end{vmatrix}. \tag{27}$$

The pertinent dimensions of the elements and the magnitudes of various other quantities are taken as in ref. [3].

Fins '1' and '4':

$$b^{(1)} = b^{(4)} = 6.34 \text{ mm}, \quad \delta^{(1)} = \delta^{(4)} = 0.152 \text{ mm}.$$

Fins '2' and '3':

$$b^{(2)} = b^{(3)} = 1.16 \text{ mm}, \quad \delta^{(2)} = \delta^{(3)} = 0.254 \text{ mm}.$$

Depth of the array: $l = 0.3048$ m.

$k = 173 \text{ W m}^{-1} \text{K}^{-1}$, $h = 56.77 \text{ W m}^{-2} \text{K}^{-1}$.

Heat flow rates: $Q_1 = 2.93$ W, $Q_5 = 2.344$ W.

There is no heat transfer at nodes '2' and '4', hence we set $Q_2 = Q_4 = 0$.

The solution for the temperature in excess of the surrounding temperature was obtained by utilizing the numerical values given above and the results are: $\theta_1 = 11.2010$, $\theta_2 = 9.7652$, $\theta_3 = 9.7825$, $\theta_4 = 9.7652$ and $\theta_5 = 10.7570$, where the subscripts correspond to the node number.

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SUR LA RESOLUTION DU TRANSFERT THERMIQUE A TRAVERS UN ARRANGEMENT DE SURFACES ETENDUES

Résumé—On présente une méthode générale d'analyse pour le transfert thermique à travers un arrangement de surfaces contenant un nombre quelconque d'ailettes. En traitant chaque ailette dans l'arrangement comme un élément fini dont les caractéristiques sont à déterminer par la résolution d'un problème d'ailette isolée monodimensionnelle, l'analyse du problème est réduite à la solution d'un système d'équations algébriques. Dans ce système, les éléments de la matrice sont reliés aux solutions fondamentales de l'équation de l'ailette. Des expressions explicites sont présentées pour la détermination des éléments de la matrice. L'application est illustrée par un exemple.

ZUR LÖSUNG DES WÄRMEÜBERGANGS-PROBLEMS AN EINEM FELD BERIPPTER OBERFLÄCHEN

Zusammenfassung—Eine allgemeine Methode zur Untersuchung des Wärmeübergangs an einem Feld berippter Oberflächen mit beliebiger Anzahl von Rippen wird mitgeteilt. Durch Behandlung jeder Rippe des Feldes als finites Element, dessen Eigenschaften aus der Lösung des eindimensionalen Einzelrippen-Problems ermittelt werden, vereinfacht sich die Untersuchung auf die Lösung eines algebraischen Gleichungssystems. In diesem System sind die Elemente der Koeffizienten-Matrix auf die Grundlösung der Rippengleichung bezogen. Explizite Ausdrücke zur Bestimmung der Elemente der Koeffizienten-Matrix werden angegeben. Die Anwendung wird anhand eines Beispiels erläutert.

О РЕШЕНИИ ЗАДАЧИ ПЕРЕНОСА ТЕПЛА ЧЕРЕЗ РАЗВИТЫЕ ПОВЕРХНОСТИ

Аннотация—Дан общий метод анализа переноса тепла через развитые поверхности, содержащие любое число ребер. При рассмотрении каждого ребра как конечного элемента, характеристики которого определяются решением одномерной задачи для единичного ребра, анализ задачи сводится к решению системы алгебраических уравнений. В этой системе элементы матрицы коэффициентов отнесены к фундаментальным решениям уравнения для ребра. Представлены выражения в явном виде для определения элементов матрицы коэффициентов, использование которых иллюстрируется на примере.